THE PROBLEM OF THE AMBER SIGNAL LIGHT IN TRAFFIC FLOW

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A theoretical analysis and observations of the behavior of motorists confronted by an amber signal light are presented. A discussion is given of the following problem: when confronted with an improperly timed amber light phase a motorist may find himself, at the moment the amber phase commences, in the predicament of being too close to the intersection to stop safely or comfortably and yet too far from it to pass completely through the intersection before the red signal commences. The influence on this problem of the speed of approach to the intersection is analyzed. Criteria are presented for the design of amber signal light phases through whose use such 'dilemma zones' can be avoided, in the interest of over-all safety at intersections.

WE LIVE in a difficult and increasingly complex world where man-made systems, man-made laws and human behavior are not always compatible. This paper deals with a problem peculiar to our present civilization, for which a satisfactory solution based on existing information and analysis is not available. The problem in question is that of the amber signal light in traffic flow.

Undoubtedly everyone has observed at some time or other the occurrence of a driver crossing an intersection partly during the red phase of the signal cycle. There are few of us who have not frequently been faced with such a decision-making situation when the amber signal light first appears, namely, whether to stop too quickly (and perhaps come to rest partly within the intersection) or to chance going through the intersection, possibly during the red light phase. In view of this situation we were led to consider the following problem: can criteria presently employed in setting the duration of the amber signal light at intersections lead to a situation wherein a motorist driving along a road within the legal speed limit finds himself, when the green signal turns to amber, in the predicament of being too close to the intersection to stop safely and comfortably and yet too far from it to pass through, before the signal changes to red, without exceeding the speed limit? From experience we feel that a problem exists, and we ask if it is feasible to construct a signal light system such that the characteristics of a driver and his car, the geometry

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of the road and intersection, and the law are all compatible with one another.

Some thought has already been devoted to this question[1,2] but it is our opinion that the problem at hand does not appear to have been thought through deeply enough as a problem in operations research nor does it appear to have been supported adequately by published observational and experimental data. It is our intention in this paper to contribute toward the understanding of this situation. First, we derive and discuss some simple relations between car speed, driver decision and reaction time, the parameters of the road and intersection, and the duration of the amber signal light. The results of measurements of the duration of amber signal lights, driver decision plus reaction time, and other parameters entering into the theoretical discussion are next presented. Finally, we discuss the experimental results in the light of theory and the traffic codes of cities and towns throughout the country.

We are well aware that there may be practical difficulties involved in incorporating the results and conclusions of an analysis such as ours into the practical planning of traffic systems, and we do not consider such problems here. It is our hope, rather, that in pointing out the existence and nature of the amber-signal-light problem we may stimulate others to pursue it further and make certain that the driver is confronted with a solvable decision problem. We are, of course, also motivated by the desire to contribute effectively toward the improvement of over-all driver safety and, in this case specifically, safety at intersections.

ANALYTICAL CONSIDERATIONS

WE CONSIDER the traffic situation depicted in Fig. 1, in which a car traveling at a constant speed v_0 toward an intersection is at a distance x from the intersection when the amber phase commences. The driver is then faced with two alternatives. He must either decelerate and bring his car to a stop before entering the intersection or go through the intersection, accelerating if necessary, and complete his crossing before the signal turns red. In these cases his acceleration or deceleration will begin at a time δ_1 or δ_2 after the initiation of the amber phase, respectively. These time intervals δ_i measure the reaction time-lag of the driver-car complex as well as the decision-making time of the driver.

In order to carry out a mathematical investigation of the problem we assume a constant acceleration a_1 in the case of crossing the intersection, or a constant deceleration a_2 in the case of stopping before entering the intersection. If, furthermore, the effective width of the intersection is denoted by w, the length of the car by L and the duration of the amber phase by τ, the following relations can be derived:

1. If the driver is to come to a complete stop before entering the intersection, we find that

$$(x-v_0\,\delta_2) \geqq v_0^2/2a_2. \tag{1}$$

2. If the driver is to clear the intersection completely before the light turns red, we must have

$$(x+w+L-v_0\,\delta_1) \leqq v_0\,(\tau-\delta_1)+\tfrac{1}{2}\,a_1\,(\tau-\delta_1)^2. \tag{2}$$

It is to be noted that the effective width, w, used in the preceding equation is meant to denote the approximate distance between a painted stopping line or a building line and a 'clearing line' whose position is necessarily somewhat indefinite because of the geometry of real intersections.

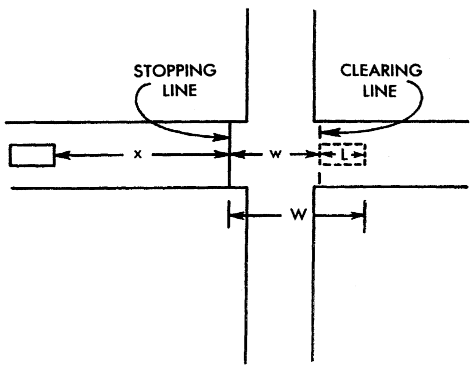

Fig. 1. Geometry of an intersection showing distances to be covered by a car of length L in the two alternative cases of going through and stopping before the intersection.

Equations (1) and (2) can be used for the discussion of the two alternatives and their ramifications. Thus, solving equation (1) for a_2 we obtain, assuming the equality sign,

$$a_2=\tfrac{1}{2}\,v_0^2/(x-v_0\,\delta_2). \tag{3}$$

Equation (3) gives the (constant) deceleration needed in order to bring the car to a stop just before the intersection as a function of the distance of the car from the intersection at the initiation of the amber phase. We see that a_2 becomes infinite for $x=v_0\,\delta_2$, as it must. However, even for values of x greater than $v_0\,\delta_2$, the deceleration given by (3), while finite, may be so large as to be uncomfortable to the driver and his passengers, or may be unsafe under the prevailing road conditions, or even physically impossible. Therefore, assuming the existence of a maximum deceleration a_2^* by which the car can be brought to a stop before the intersection safely and comfortably, equation (1) defines a 'critical distance', namely,

$$x_c=v_0\,\delta_2+v_0^2/2a_2^*. \tag{4}$$

If $x>x_c$ the car can be stopped before the intersection, but if $x<x_c$ it will be uncomfortable, unsafe, or impossible to stop. We note that this critical distance is independent of the duration of the amber phase, τ, and depends only on the characteristics of the driver-car complex. The required deceleration is plotted versus distance in Fig. 2.

Turning, now, to the second alternative, namely, going through the

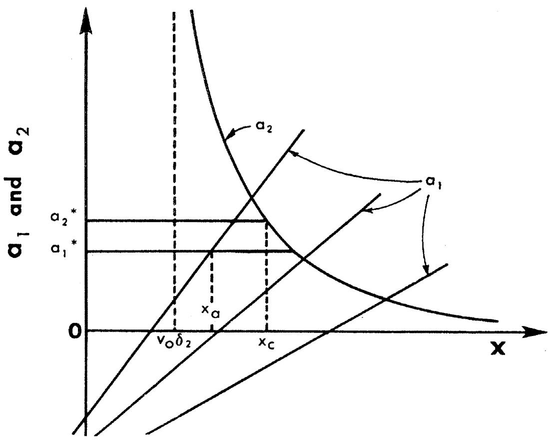

Fig. 2. Variation of the deceleration required in order to stop before the intersection, a_2, or the acceleration required to clear the intersection, a_1, versus the distance from the intersection, x. The x-intercept of the a_1 versus x lines defines a distance x_0 which is the maximum distance, apart from the width of the intersection and the length of car, which can be covered without acceleration during the amber phase.

intersection, we solve equation (2) for a_1, assuming the equality sign, and obtain

$$a_1=2\,x/(\tau-\delta_1)^2+2\,(w+L-v_0\,\tau)/(\tau-\delta_1)^2. \qquad (5)$$

Equation (5) gives the (constant) acceleration needed in order that the car may clear the intersection just as the signal turns red, as a function of the distance x of the car from the intersection at the start of the amber phase. For various values of the parameters involved, equation (5) represents a family of straight lines in the x,a_1-plane with slope

$$da_1/dx=2/(\tau-\delta_1)^2, \qquad (6)$$

and intercept on the x-axis,

$$x_0=v_0\tau-(w+L). \qquad (7)$$

The quantity x_0 is the maximum distance the car can be from the intersection at the start of the amber phase and still clear the intersection without acceleration during the amber phase. The position of x_0 with respect to x_c, and the character of the line represented by equation (5), determine whether or not the duration of the amber phase has been adequately designed, taking into account the requirements of the law and the physical 'boundary conditions' of the problem. Thus, if $x_0 > x_c$, the driver,

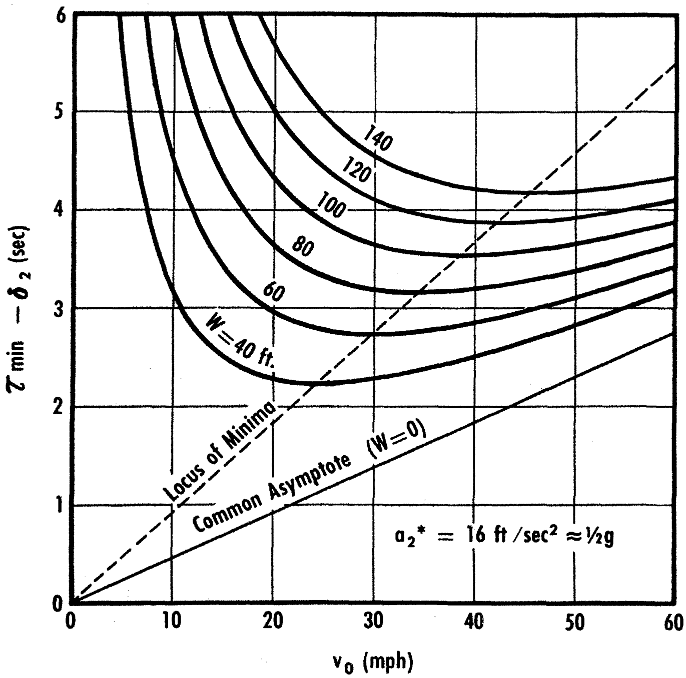

Fig. 3. Variation of the minimum amber-phase duration, τ_{min}, which is required in order that there be no dilemma zone, versus constant approach speed, v_0, for various intersection widths plus car length, W. (The constant deceleration is assumed to be 16 ft/sec^2.)

once past the critical distance x_c, can clear the intersection before the signal turns red. If, however, $x_0 < x_c$, a driver at a distance x from the intersection such that $x_0 < x < x_c$ will find himself in a very awkward position if the amber phase begins at that moment. He cannot stop safely and hence he has to attempt to go through the intersection. From Fig. 2 we see that he can achieve this only by accelerating. If, however, v_0 happens to be the maximum allowable speed, the driver will find himself in the following predicament. He can neither bring his car to a stop safely nor can he go through the intersection before the signal turns red without violating the speed limit.

There is an even worse possibility, which is realized for even shorter values of τ. This is the case where $x_0 < x_c$ and the slope da_1/dx is sufficiently large that the line represented by equation (5) intersects a line $a_1 = a_1^*$, where a_1^* is a maximum possible acceleration, at a point which has an abscissa x_a smaller than x_c. Then, for $x_a < x < x_c$, a driver cannot stop safely and he cannot clear the intersection before the initiation of the red light phase even if he is willing to utilize all the power resources of his car while violating the speed limit.

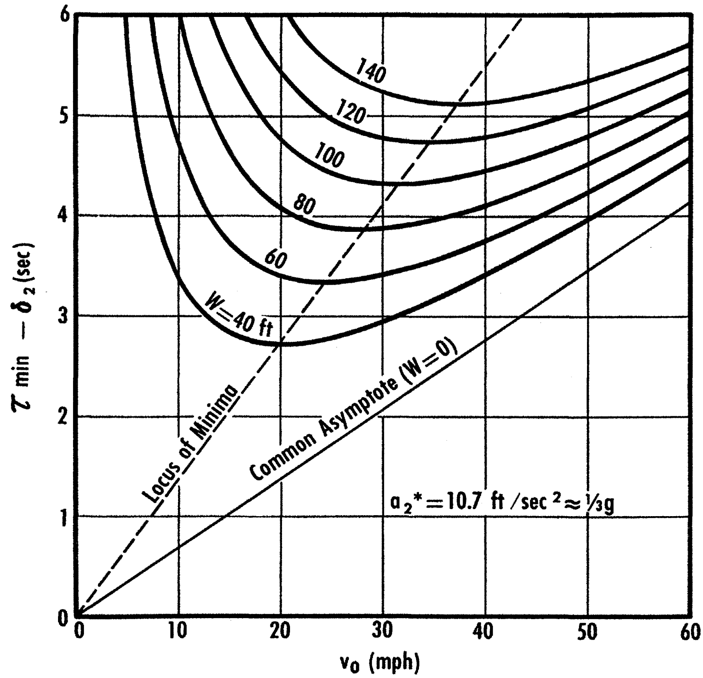

Fig. 4. Variation of the minimum amber-phase duration, $\tau_{\min}$, which is required in order that there be no dilemma zone, versus constant approach speed, v_0, for various intersection widths plus car length, W. (The constant deceleration is assumed to be 10.7 ft/sec^2.)

It may be pointed out that this maximum possible acceleration depends on the approach velocity v_0. It is well known that the higher the velocity of a car the lower its accelerating capability. Thus an average good car can have an acceleration of as much as ½ g starting from rest, but only about 0.08 g when traveling at 65 mi/hr.† (Note that g is the earth's gravitational acceleration.)

Let us now discuss the design of the duration of the amber phase. From the graphical representation of Fig. 2, we see that the minimum

† We are indebted to Mr. Joseph Bidwell for furnishing us with the experimental data on the accelerating capability of a car as a function of its speed.

TABLE I

COMPARISON OF OBSERVED AND CALCULATED AMBER-PHASE DURATIONS

Street	Cross street	Speed limit (mi/hr)	Approximate effective width of intersection	Duration of amber phase	Theoretical τ_{min}: eq. (5)[a] $a_2^*=10.7$ ft/sec² $\delta_2=1.14$ sec	$a_2^*=10.7$ ft/sec² $\delta_2=0.75$ sec	$a_2^*=16$ ft/sec² $\delta_2=1.14$ sec	$a_2^*=16$ ft/sec² $\delta_2=0.75$ sec
South of Main	Catalpa	25	60	2.7[b]	4.91	4.52	4.33	3.94
North on Mound	Chicago	30	75	3.4	5.25	4.86	4.56	4.17
East on Chicago	Van Dyke	30	80	4.0	5.36	4.97	4.67	4.28
North on Woodward	Calvert	30	—	3.6	—	—	—	—
East on 11 Mile	Van Dyke	35	55	3.4	4.90	4.51	4.10	3.71
West on 14 Mile	Southfield	35	60	6.8	5.00	4.61	4.20	3.81
South on Woodward	9 Mile	35	80 to 120	4.5	5.39 6.16	5.00 5.77	4.59 5.36	4.20 4.97
North on Woodward	Savannah	35	65	3.85	5.10	4.71	4.30	3.91
North on Mound	13 Mile	40	50	3.6	5.00	4.61	4.09	3.70
West on Chicago	Van Dyke	40	80	4.0	5.51	5.12	4.60	4.21
West on 8 Mile	Ryan	40	70	3.9	5.34	4.95	4.43	4.04
North on Van Dyke	12 Mile	40	80	4.1	5.51	5.12	4.60	4.21
East on 12 Mile	Van Dyke	45	65	4.0	5.44	5.05	4.41	4.02
North on Woodward	11 Mile	45	80	3.44	5.67	5.28	4.64	4.25
North on Woodward	Lincoln	45	75	3.75	5.59	5.20	4.56	4.17
South on Van Dyke	Chicago	50	70	3.8	5.74	5.35	4.60	4.21

[a] Two values of the time lag δ_2 were assumed. One of them is the observed average 1.14 sec and the other a lag of 0.75 sec frequency assumed as a minimum. A car length was taken as 15 ft to be conservative. Two values for the maximum deceleration a_2^* were assumed. One of them is equal to $\frac{1}{3}g$ which is feasible but is a fairly high deceleration not desirable in normal driving. The other one is equal to $\frac{1}{2}g$, which corresponds to a very hard stop. (Note that 0.6 g is about the absolute maximum deceleration under ideal conditions.)

[b] The amber phase here was measured at about 2.1 sec prior to a modification in the signal cycle. We have been informed of an even shorter amber phase of only about 1.5-sec duration at an intersection in California where an individual received a ticket for being in this intersection on the red signal.

amber-light duration, denoted by τ_{min}, which guarantees the safe execution of either one of the alternatives of stopping or going through the intersection without accelerating, corresponds to $x_0=x_c$. Hence

$$\tau_{min}=(x+w+L)/v_0, \tag{8}$$

and, using equation (4),

$$\tau_{min}=\delta_2+\tfrac{1}{2}\, v_0/a_2^*+(w+L)/v_0. \tag{9}$$

A simple numerical example will show the magnitude of the quantities involved. Assuming $v_0=45$ mi/hr$=66$ ft/sec, $a_2^*=0.5$ $g\approx 16$ ft/sec^2, $\delta_2=1$ sec, $w=65$ ft, and $L=15$ ft, we find $x_c=202$ ft and $\tau_{\min}=4.28$ sec.

It may be noted that the length of the car, L, is added to the effective width of the intersection, w, in order to determine the length of travel through the intersection. The length of the car contributes the quantity

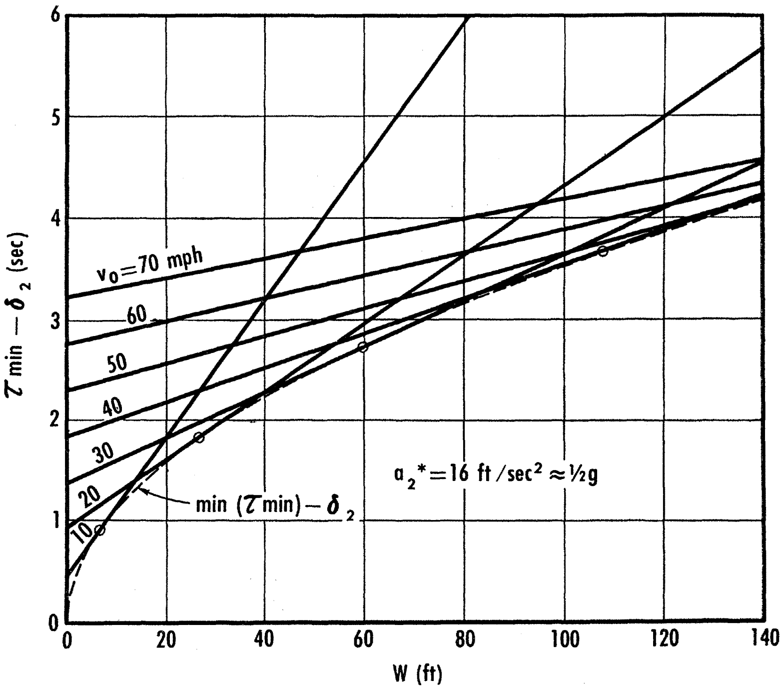

Fig. 5. Variation of the minimum amber-phase duration, $\tau_{\min}$, which is required in order that there be no dilemma zone, versus the intersection width plus car length, W, for various values of the constant approach speed, v_0. (The constant deceleration is assumed to be 16 ft/sec^2.)

L/v_0 in the computation of $\tau_{\min}$. This means that the required $\tau_{\min}$ is substantially longer for vehicles such as long trucks, buses, or vehicles with trailers, even assuming that these vehicles can stop with the same maximum deceleration a_2^* as shorter ones. One may retort that traffic signals should not be designed for these 'unusual' cases. However, these unusual vehicles *are* allowed on the highways, and if the design of the amber phase does not take them into account then the questions raised in the introduction regarding the compatibility of law and physical characteristics become even more acute.

Returning now to the expression for $\tau_{\min}$ given in equation (9), we use this result to plot $\tau_{\min}$ versus v_0 in Figs. 3 and 4 for various values of the parameter

$$W = w + L \qquad (10)$$

and for two values of the maximum deceleration a_2^*, namely, $\frac{1}{2}\,g$ and $\frac{1}{3}\,g$. (For comments on the magnitude of these decelerations, see the first

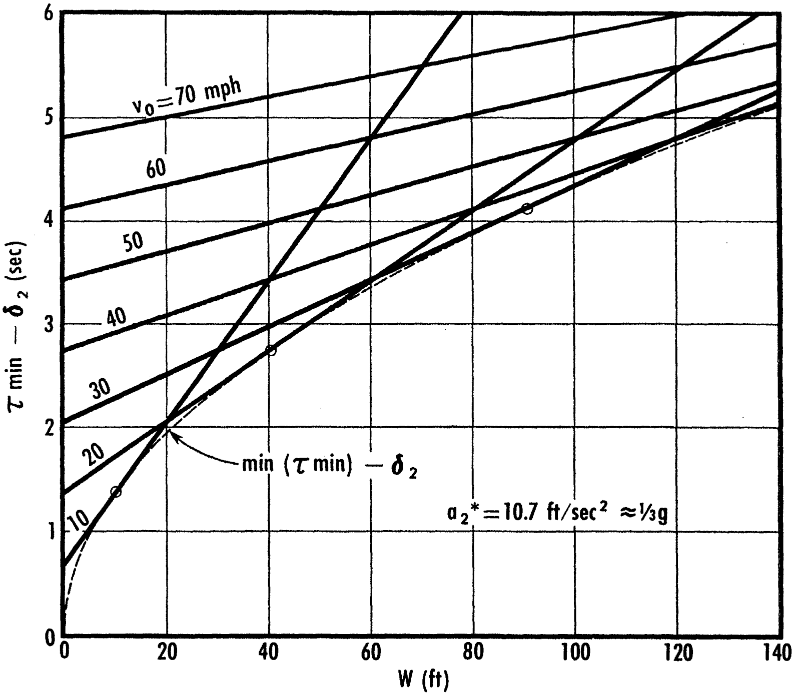

Fig. 6. Variation of the minimum amber-phase duration, $\tau_{\min}$, which is required in order that there be no dilemma zone, versus the intersection width plus car length, W, for various values of the constant approach speed, v_0. (The constant deceleration is assumed to be 10.7 ft/sec.)

footnote in Table I, as well as reference 2, p. 68.) The minima of the various curves correspond to values of the approach velocity v_0, assumed equal to the speed limit, which would minimize $\tau_{\min}$ for a given value of W. From equation (9) we have

$$\partial\tau_{\min}/\partial v_0 = 1/2a_2^* - W/v_0^2, \qquad (11)$$

and $\partial\tau_{\min}/\partial v_0 = 0$ for $\qquad v_0 = \sqrt{2\,a_2^*\,W}. \qquad (12)$

Hence the absolute minimum length of the amber phase is given by

$$\min(\tau_{\min}) = \delta_2 + \sqrt{2\,W/a_2^*}. \qquad (13)$$

Figures 5 and 6 contain plots of $(\tau_{\min}-\delta_2)$ versus W for different values of the approach velocity v_0, and for the same two values of a_2^* as in Figs. 3 and 4. Equation (9) yields a family of straight lines in the plane $(\tau_{\min}-\delta_2)$ versus W. The envelope of these lines corresponds to $\min(\tau_{\min})$ as given by equation (13).

The foregoing discussion is illustrated in Fig. 7, where each of the two shaded zones precludes one of the two alternatives of stopping or going through the intersection. Thus, a car at a distance from the intersection smaller than x_c cannot stop safely, whereas a car at a distance greater

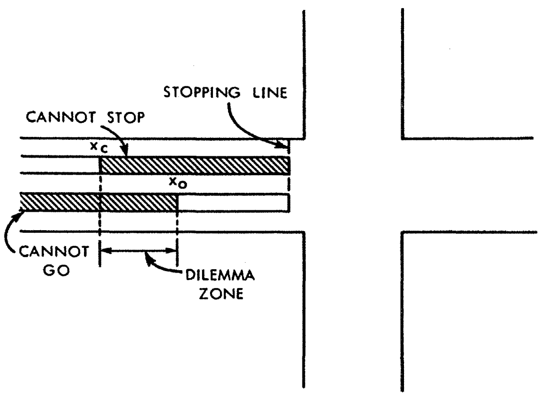

Fig. 7. Schematic diagram showing the 'dilemma zone' near an intersection.

than x_0 cannot go through the intersection without accelerating before the light turns red.

As mentioned already, when $x_0<x_c$ the driver is in trouble if he finds himself in the region $x_0<x<x_c$, which in the sequel will be referred to as the 'dilemma zone.'

The preceding arguments have been established on the assumption that the approach speed of the motorist is equal to the speed limit so that he cannot accelerate to clear the intersection without exceeding the speed limit. It is possible, however, that even if the amber phase is improperly set so that a dilemma zone exists for an approach speed equal to the speed limit, a motorist may, under certain circumstances, avoid encountering such a dilemma zone if his approach speed is smaller than the speed limit. This is so because the critical distance, x_c, decreases rapidly as the approach speed decreases. On the other hand, if the driver is at a distance from the intersection slightly larger than this reduced x_c when the amber-light phase begins he may be able, under certain circumstances, to clear the intersection within this phase by accelerating until he has reached the speed limit and then proceeding through the intersection at this speed. An example of this case is illustrated in Fig. 10, which is discussed a little later.

If we assume that the driver's acceleration from v_0 to v_l (the speed

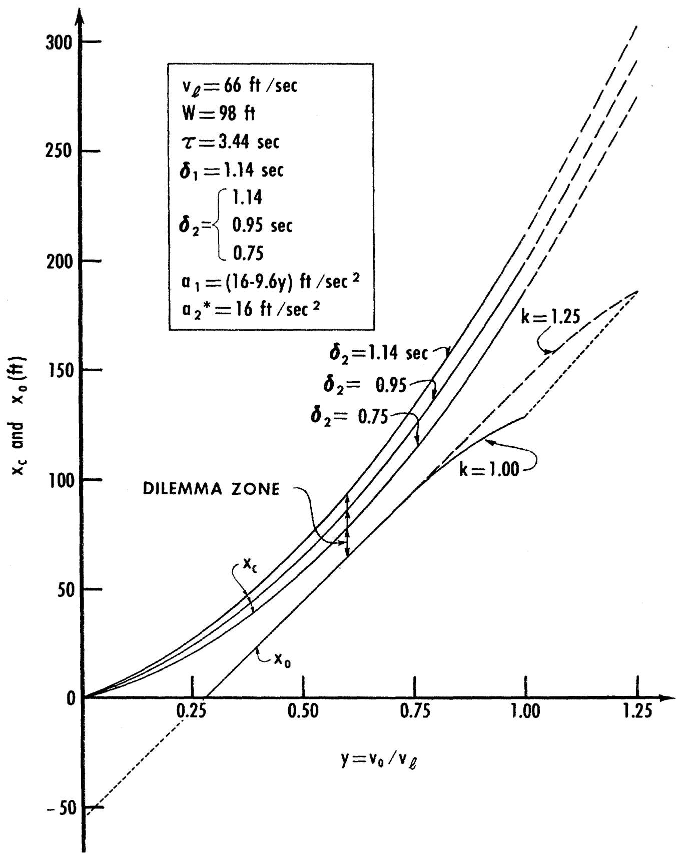

Fig. 8. Northbound on Woodward Avenue at 11 Mile Road. Variation of the critical distance, x_c, and the maximum distance which can be covered within the amber phase duration, x_0, versus the ratio of the approach speed to the speed limit, $y = v_0/v_l$. It is assumed that in crossing the intersection the car may accelerate up to a speed not in excess of kv_l.

limit) is constant and equal to a_1, the equation which replaces equation (2) is

$$x_0 = \begin{cases} v_0\,\delta_1 - W + (v_l^2 - v_0^2)/2a_1 + v_l\,[\tau - \delta_1 - (v_l - v_0)/a_1] \\ \qquad\qquad\qquad \text{for} \quad \tau \geqq \delta_1 + (v_l - v_0)/a_1, \\ v_0\,\delta_1 - W + v_0\,(\tau - \delta_1) + (\tfrac{1}{2}\,a_1)\,(\tau - \delta_1)^2 \\ \qquad\qquad\qquad \text{for} \quad \tau \leqq \delta_1 + (v_l - v_0)/a_1, \end{cases} \tag{14}$$

where W is given by (10) and x_0 is the distance of the car from the intersection at the moment the amber phase commences. It is assumed that the car just clears the intersection before the light turns red. Rewriting (14) to give x_0 as a function of $y=v_0/v_l$, where $0\leqq y\leqq 1$, we obtain

$$x_0=\begin{cases} -W+v_l\,\tau-v_l\,\delta_1\,(1-y)-(v_l^2/2a_1)(1-y)^2 & \\ \qquad\qquad\qquad\qquad\qquad \text{for} \quad \tau\geqq\delta_1-(v_l/\alpha_1)(1-y), & \\ -W+\tfrac{1}{2}\,a_1(\tau-\delta_1)^2+v_l\,\tau y \qquad \text{for} \quad \tau\leqq\delta_1-(v_l/a_1)(1-y). & \end{cases} \tag{15}$$

Equation (1) remains unchanged, so that

$$x_c=\delta_2\,v_l\,y+(v_l^2/2a_2^*)\,y^2. \tag{16}$$

For simplicity we assume that $\delta_1=\delta_2=$ 1.14 sec (*see* the following section), while $a_2^*=\tfrac{1}{2}\,g=16$ ft/sec^2. The (constant) acceleration a_1 is, however, a function of the car speed at the moment when the car begins to accelerate. An analytic expression for this speed dependence of a_1, which fits the experimental data adequately enough for our purposes is

$$a_1(v_0)=\begin{cases}(16-0.145\,v_0)\text{ ft/sec}^2 & \text{for} \quad 0\leqq v_0\leqq 110\text{ ft/sec},\\ 0 & \text{for} \quad v_0 \quad >110\text{ ft/sec},\end{cases} \tag{17}$$

where v_0 is given in ft/sec. We assume, for simplicity, that a car traveling at an approach speed v_0 can maintain a constant acceleration a_1, as given by equation (17), for a length of time of the order of τ. It should be noted that there are marked differences in the dynamic characteristics of various cars with regard to acceleration. The preceding equation gives an acceleration which is on the high side and is applicable to the high-powered modern car. Low-powered cars develop considerably lower accelerations, particularly at high speeds. If one were to assume lower accelerations, the problem of the dilemma zone would be accentuated.

Using equations (15), (16), and (17), we have plotted x_0 and x_c as functions of y for three different intersections in Figs. 8, 9, and 10.

The curve for x_0 has a straight segment, corresponding to the second expression in (15), and a curved segment corresponding to the first expression. These two segments are tangent at the point y_t satisfying the equation

$$1-y_t-(\tau-\delta_1)(a_1/v_l)=0. \tag{18}$$

Hence, in view of (17), we have

$$y_t=[1-16\,(\tau-\delta_1)/v_l]/[1-0.145\,(\tau-\delta_1)], \tag{19}$$

where speeds are given in ft/sec and times in seconds.

From Fig. 7 we see that there is no dilemma zone if $x_0 > x_c$; of the situations depicted in Figs. 8–10 we see that in only one case, namely, that shown in Fig. 10, is there an absence of a dilemma zone, and this is so only for $0.15 < y < 0.57$. This means that at this particular intersection a car traveling at the speed limit of 65 mi/hr would encounter a dilemma zone of 106 ft, approximately six car-lengths, at a distance of 286 ft from the

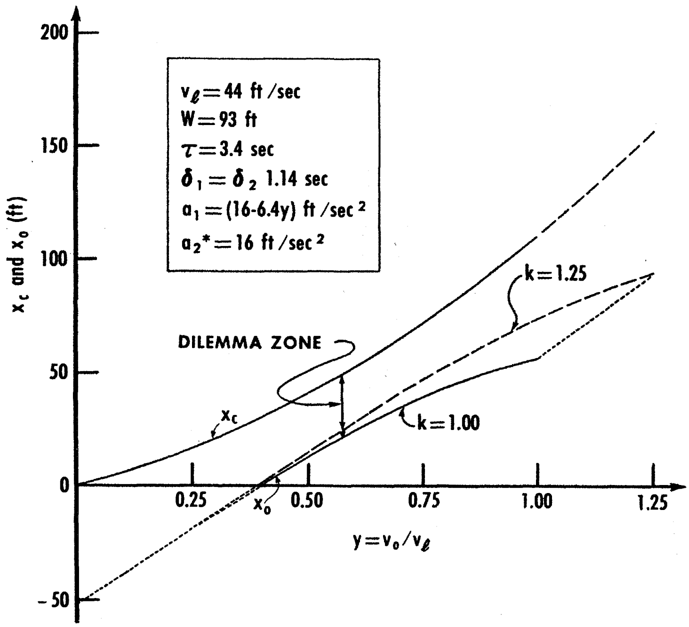

Fig. 9. Northbound on Mound Road at Chicago Road. Variation of the critical distance, x_c, and the maximum distance which can be covered within the amber-phase duration, x_0, versus the ratio of the approach speed to the speed limit, $y = v_0/v_l$. It is assumed that in crossing the intersection the car may accelerate up to a speed not in excess of kv_l.

intersection. On the other hand, if the speed of the car is 37 mi/hr or lower, no such zone exists. It need hardly be pointed out that under ordinary driving conditions a speed of 37 mi/hr on a highway with a 65 mi/hr-maximum is unrealistic, and quite possibly dangerous.

From the preceding discussion we ascertain that if one were to assume, for low-powered cars, accelerations lower than those given by (17), the values of x_0 would be reduced considerably and the dilemma zones increased in the entire range $0 \leqq y \leqq 1$.

Approaching an intersection at a speed lower than the speed limit is one facet of defensive driving. It is seen from the preceding discussion that this in itself is not always sufficient to obviate the dilemma-zone problem. Another facet of such defensive driving consists of the maneuver of coasting toward the signal light with one's foot readied on the brake. The advantage, in this case, which comes from shortening the reaction

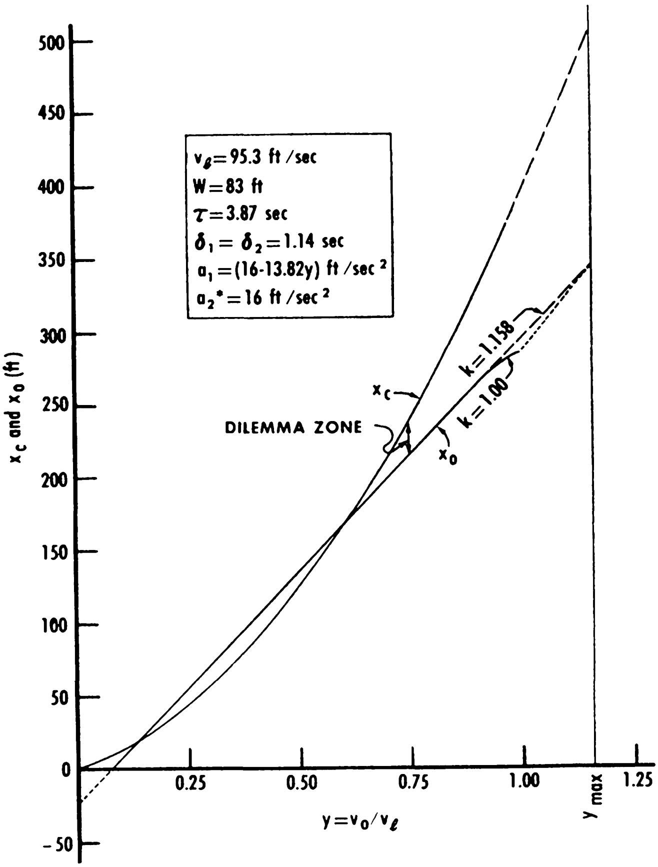

Fig. 10. Northbound on Stephenson Highway at 15 Mile Road. Variation of the critical distance, x_c, and the maximum distance which can be covered within the amber-phase duration, x_0, versus the ratio of the approach speed to the speed limit, $y = v_0/v_l$. It is assumed that in crossing the intersection the car may accelerate up to a speed not in excess of kv_l. The value of $k = 1.157$ corresponds to the maximum speed of 110 ft/sec according to equation (17).

time, is reflected in a decrease of the critical distance x_c. The improvement, which is by no means an absolute cure, can be seen from the curves plotted in Fig. 8 for two values of δ_2 other than the observed average. Such defensive driving, however, should be used with discrimination and great caution when approaching intersections in a high-density traffic pattern since it may induce a rear-end collision—a prominent type of accident in traffic today.

Many drivers take the attitude that there is nothing sacred about the speed limit! Suppose one, starting with an initial speed $v_0 = yv_l$, where v_l is again the official speed limit, accelerates to a final speed equal to or less than v_l' given by

$$v_l' = kv_l. \qquad (k>1) \quad (20)$$

The analysis already carried out can be applied to this case on the assumption that the 'effective speed limit' is $v_l' = kv_l$ and the initial speed

$$v_0 = y'v_l' = (y/k)\, v_l'. \qquad (0 \leqq y' \leqq 1) \quad (21)$$

The x_c versus y curve obviously does not change. The ordinate of the x_0 versus y curve at $y'=1$, i.e., at $y=k$, is

$$x_0^* = -W + v_l\, \tau k. \qquad (22)$$

In Figs. 8 and 9 we have plotted with dashed lines the curves of x_0 corresponding to 'effective speed limits' equal to 1.25 v_l (i.e., $k=1.25$). Similarly in Fig. 10 we have plotted with a dashed line the curve of x_0 for $k=1.158$. This value of k corresponds to an 'effective speed limit' equal to the assumed maximum possible speed of 110 ft/sec (75 mi/hr), according to equations (17). Again, these curves are made up of two segments, one straight and one curved, which are tangent at the point

$$y_t' = [k - 16\ (\tau - \delta_1)/v_l]/[1 - 0.145\ (\tau - \delta_1)]. \qquad (23)$$

The straight segment is an extension of the one already plotted on the basis of the second expression in (15), which is independent of the effective speed limit.

From these figures we see that even if the driver is willing to accelerate to speeds greatly in excess of the speed limit, he still cannot eliminate the dilemma zone.

With regard to the length of the dilemma zone, the following additional remark can be made on the basis of the preceding discussion. If a driver encounters a dilemma zone, the maximum possible distance of the rear bumper of his car from the clearing line of Fig. 1 at the moment the red phase commences is equal to the length of the dilemma zone. This maximum distance is realized if the driver is just past x_c when the amber phase commences. Now, if the indecision zone is greater than the effective width of the intersection plus the car length, W, the driver may even have to *enter* the intersection during the red phase. From Fig. 10 it is seen that this may happen, at the intersection under consideration, to a driver who approaches the intersection at the speed limit and does not want to exceed this limit, since in this case the dilemma zone of 106 ft is greater than $W = 83$ ft.

OBSERVATIONS

In order to compare the theoretical results of the preceding section with physical reality the following kinds of observations were carried out on

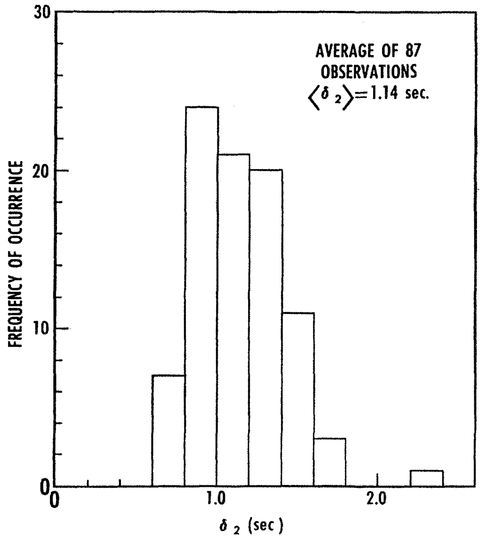

Fig. 11. Histogram showing the observed frequency of occurrence of various intervals of decision and reaction time in braking, δ_2, in a total of 87 measurements.

the manner in which people actually drive and the pattern in which amber signal light phases are in practice set:

1. Duration of amber-light phase.

2. Motorists' braking reaction time (including the decision time and the reaction time lag).

3. Average number of motorists per cycle who run through the red light.

4. The dimensions of the road and intersection together with the posted speed limit.

5. Traffic density.

6. The effective critical distance x_c.

Most of the observations were made at street intersections within about a fifteen-mile radius of the General Motors Technical Center. It was not our intention to make our data exhaustive, but we feel that enough measurements were made so that fairly definite conclusions based on them could be drawn.

We begin by presenting in Table I a sampling of the data obtained on amber-signal-light times, speed limits, and approximate intersection widths, at a number of intersections, together with theoretical values of the minimum amber-light phase, $\tau_{\min}$, calculated from equation (8) using two values of the maximum deceleration and two values of the braking reaction time.

TABLE II

OBSERVED AND CALCULATED CRITICAL DISTANCE, x_c

Street	Cross street	Speed limit	Effective x_c	Theoretical x_c ($a_2^*=0.5\ g$)
North on Woodward Avenue	Lincoln	45 mi/hr	165 ft	211 ft
West on 8 Mile Road	Ryan	40	145	174
North on Woodward Avenue	11 Mile Road	45	185	211

In measuring the drivers' braking reaction time, an observer was stationed near a given intersection at a distance somewhat greater than the estimated x_c. The observer would then arbitrarily choose a car in the interval between himself and the intersection and would measure the time

TABLE III

TRAFFIC FLOW AND PER CENT TRAFFIC-LIGHT VIOLATIONS

Street	Cross street	Number cars in intersection per cycle	Average number cars running through red signal per cycle	Per cent of cars running through red signal	Amber phase (sec)
North on Woodward Avenue	Lincoln	62.3	1.2	1.93	3.75
		53.8	0.8	1.49	3.73
West on 8 Mile Road	Ryan	42.1	0.7	1.66	3.9
North on Woodward Avenue	11 Mile Road	54.5	1.2	2.20	3.49
North on Woodward Avenue	Woodland	91.6	0.5	0.55	4.23
North on Woodward Avenue	Sylvan	95.1	0.1	0.11	4.69
North on Woodward Avenue	Webster	46.1	0.4	0.87	3.67

interval between the moment the amber signal came on and the moment when the red brake tail light flashed. The distribution of such delay times is plotted in Fig. 11 on the basis of 87 observations. The mean delay time was found to be 1.14 seconds.

The determination of an average effective x_c was carried out using the following criterion: it is the closest distance at which a car can be from the intersection, when the amber signal commences, and still be capable of stopping before entering the intersection. Measurements of this quantity were made at several intersections and the results are shown in Table II together with the theoretical values calculated from equations (4). The observed x_c was in general a little smaller than the theoretical x_c corresponding to the speed limit of the observed intersections. This was probably due to the fact that the traffic was moving, on the average, a little slower than the posted speed limit, since our observations were made during the heavy traffic of the rush hour.

Finally, we measured at a few intersections the average number of cars that ran through the red signal per signal light cycle during rush hour traffic (4:30–6:00 P.M.), together with the average number of cars that pass through the intersection per signal light cycle. These results are shown in Table III.

The preceding pertains to a single traffic light. Analogous results may be obtained for two closely spaced traffic lights, as in the case of crossing of a divided highway. However, this case is rather complicated and will not be discussed here. There are other variations to the problem of the dilemma zone such as the case of a vehicle approaching an intersection at slow speed with the intention of making a turn. This is a case of known practical difficulty and some information can be obtained from the present analysis with w taken equal to the distance traversed while turning.

Some additional data regarding the amber-light phase were obtained from three other cities, namely, Washington, D. C., Silver Spring, Maryland, and Los Angeles, California. On the average, the amber-light phases were slightly shorter in Los Angeles and slightly longer in the Washington, D. C., area, relative to those in the Detroit area. There are no significant differences, and the conclusions of this paper will apply in those areas also.

DISCUSSIONS AND CONCLUSIONS

THE *Uniform Vehicle Code* of the National Committee on Uniform Traffic Laws and Ordinances[3] gives the following definition for the purpose of the amber signal light:

> Vehicular traffic facing the signal is thereby warned that the red or 'Stop' signal will be exhibited immediately thereafter and such vehicular traffic shall not enter or be crossing the intersection when the red or 'Stop' signal is exhibited.

Most of the traffic ordinances throughout the United States that we have seen have followed this definition with slight variations such as the omission of the phrase "or be crossing (the intersection). . . ." Some ordinances make an attempt to provide an operational definition of the meaning of the amber signal with definite instructions to the driver on how to behave. A typical example of such an ordinance is the following:

Vehicular traffic facing the signal shall stop before entering the nearest crosswalk at the intersection, but if such stop cannot be made in safety, a vehicle may be driven cautiously through the intersection.

Both definitions, of course, assume that the signal has been designed properly so that the driver can behave as directed and in general can solve the decision problems he encounters. It is interesting to note that the *Manual on Uniform Traffic Control Devices for Streets and Highways*[4] makes the following statement:

Confusion has frequently arisen from the misuse of this yellow lens. When the length of yellow vehicle-clearance interval is correct, and the standard meaning above described† is generally observed, necessary functions of warning and clearing the intersection are performed by this interval.

This is a reasonable statement to which we, of course, subscribe. Our investigations show, however, that out of approximately 70 intersections studied, only one had an amber phase long enough to prevent an appreciable dilemma zone, i.e., a zone longer than about one car-length, if one assumes a 'comfortable' deceleration of $\frac{1}{3}$ g and a decision and reaction time-lag equal to our measured average of 1.14 sec. Even if one assumes the very large deceleration of $\frac{1}{2}$ g and a decision-reaction time lag of 0.75 sec, only four out of the 16 typical intersections of Table I yield a dilemma zone smaller than one car-length. Out of these four, one, namely the sixth zone in Table I, gives no dilemma zone at all and is the only such intersection observed in the area.

The fact that almost all the intersections have sizeable dilemma zones is reflected in the data of Table III, which indicate that at the intersections studied as many as two cars went through the red light per light cycle, with an average of close to one car per cycle. It is true that in none of the observed cases did there appear to be any distinct possibility of an accident. However, the fact remains that an average of eleven out of every thousand cars were very much in the middle of the intersection when the red signal started, in violation of the *Uniform Vehicle Code*. This leaves them open to the possibility of receiving a traffic citation from an assiduous police officer. We might mention here that we were rather surprised to discover a traffic ordinance that made no distinction whatsoever between the yellow and red lights. The instruction regarding both was that "Vehicular traffic facing the signal shall stop before entering the nearest crosswalk at the intersection," a requirement which is clearly impossible to obey under many circumstances. It is interesting to note that in a state-issued driver-instruction pamphlet we again find that the amber and red lights are inter-

† The standard meaning referred to is precisely that quoted above as due to the *Uniform Vehicle Code* of the National Committee on Uniform Traffic Laws and Ordinances.[4]

preted alike without regard to the operational problems considered here. The same pamphlet instructs the drivers to "... drive a reasonable speed which will allow me to stop when the amber light comes on." The analysis given in this paper clearly shows that even reduction of speed and defensive driving when approaching an intersection does not necessarily eliminate the dilemma zone problem if the amber phase is inadequate.

The problem of determining the proper duration of the amber phase of the light cycle is perhaps more difficult and complicated than may appear at first sight. In this connection we quote MATSON, SMITH, AND HURD:[1] "In urban areas where speeds are relatively low, yellow lights of about 3-sec duration are satisfactory at most locations. At rural, high-speed locations where stopping time may have a duration of 5 to 8 sec, road users tend to attempt to clear the intersection rather than stop. Five seconds is probably a practical maximum yellow duration in such location."

We are aware of the fact that traffic engineers are inclined to shorten the amber phase for various reasons. One of them, probably one of the most important ones, is their conviction, undoubtedly substantiated, that drivers are inclined to ignore a long amber phase and treat it as merely a continuation of the green phase. They believe that as many drivers, if not more, will go through the red light when the amber phase is too long, as will do so if it is too short. However, we believe that it is the duty of the traffic engineers and the drafters of traffic ordinances to present the average, honest, driver with a solvable decision problem. As it stands now, a driver who is in the middle of an intersection when the red light comes on may not be a deliberate violator, but may be the victim of an improperly designed light cycle. It is true that accidents are in general prevented because of some delay of approach of the cross traffic and also by the judicious use of overlapping red cycles. This fact, however, does not release the unwilling violator from the legal responsibility which may become alarming in the case of an accident. On the other hand, with an adequate amber phase it would be easier to separate the violators from the nonviolators, insofar as traffic is concerned.

We believe that a correct resolution of this problem may be found in one of the following alternatives:

1. Design the amber-light phase according to some realistic criteria in order to guarantee that a driver can always be in a position to obey the law.

2. If the amber-light phases are to be kept short relative to criteria such as determined herein, it may be desirable to state the vehicle code in such a way as to make it compatible with the driver, car, road, and signal characteristics.

In either case it would be very advisable to educate both the driving public and the law-enforcing agencies as to the exact operational definition of the amber light. Needless to say, the fewer the variations of traffic ordinances in this respect, from one locality to another, the fewer the chances of confusion. We wish to re-emphasize our hope that a well-thought-out and operationally sound traffic and enforcement system, together with the healthy driver attitudes of a properly educated public, will promote safer and more efficient driving conditions.

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